CRITERION FOR GENERATION OF WINDS FROM MAGNETIZED ACCRETION DISKS

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ABSTRACT

An analytic model is proposed for non-radiating accretion flows accompanied by up or down winds in a global magnetic field. Physical quantities in this model solution are written in variable-separated forms, and their radial parts are simple power law functions including one parameter for wind strength. Several, mathematically equivalent but physically different expressions of the criterion for wind generation are obtained. It is suggested also that the generation of wind is a consequence of the intervention of some mechanism that redistributes the locally available gravitational energy, and that the Bernoulli sum can be a good indicator of the existence of such mechanisms.

Subject headings: accretion, accretion disks—magnetohydrodynamics: MHD —galaxies: nuclei, jets

1. INTRODUCTION

Blandford & Payne (1982) have derived a criterion for the generation of centrifugal winds from magnetized accretion disks. It has been shown in their self-similar solution that a centrifugal wind appears if the inclination of the poloidal magnetic field lines penetrating a Keplerian accretion disk makes an angle of less than 60° with the disk surface. Although this result is intuitively very understandable, in a more general situation that does not satisfy self similarity, this kind of criterion becomes meaningless because the inclination angle may be different at different locations even along a stream line. Therefore, we need to seek for physically more essential expressions of the criterion for the generation of winds from accretion disks, in order to improve the understanding of such processes (for general reviews of the wind and jet theories, see e.g., Begelman, Blandford & Rees 1984; Ferrari 1998; Livio 1999).

In relation to recent development of the theories of optically thin ADAFs (advection-dominated accretion flows: see e.g., Ichimaru 1977; Narayan & Yi 1994, 1995; Abramowicz et al. 1995; Chen et al. 1995), the sign of the Bernoulli sum has drawn considerable attention (and has caused also confusion) as a possible indicator of the presence of unbounded outflows such as winds or jets. The sum consists of the gravitational energy, kinetic energy and enthalpy, all per unit mass of a fluid.

Narayan & Yi (1994, 1995) have shown that the sign of the Bernoulli sum is necessarily positive in their self-similar ADAF solution, and argued that this is a genuine property of general ADAFs. They have suggested also that a flow with positive Bernoulli sum (and hence ADAFs, specifically) can easily drive winds and jets. Further developed and popularized by Blandford & Begelman (1999) in their influential "ADIOS" (adiabatic inflow-outflow solution) paper, these suggestions have been widely accepted. The latter paper stresses and demonstrates the necessity for including winds or jets in constructing a

satisfactory theoretical model of inefficiently radiating accretion flows.

Meanwhile, several authors have demonstrated that the positivity of the Bernoulli sum is not a genuine property of ADAFs. As shown by Nakamura (1998), the analytic solution of Honma (1996) that includes a diffusion cooling by turbulence has negative sign at least in a special case of $\gamma = 5/3$, where γ is the index of polytrope. Further it has been shown that, under the influence of convective cooling (Igumenshchev & Abramowicz 2000) and of the inner and outer boundary conditions (Abramowicz, Lasota, & Igumenshchev 2000), low viscosity ADAFs (with $\alpha < 0.3$, where α is the viscosity parameter) have negative sums. This result agrees with all of the subsequent 1-D, 2-D and 3-D numerical simulations by Abramowicz and his collaborators. For inviscid ADAFs, the no-wind solution in a global magnetic field (Kaburaki 2000, hereafter K00) shows that the Bernoulli sum is always zero within the adopted approximation.

Another point to be mentioned is the claim by Abramowicz, Lasota, & Igumenshchev (2000) that a positive Bernoulli sum is only a necessary but not the sufficient condition for unbounded outflows, as shown for inviscid, non-magnetic fluids. They also argue that the best example in which a positive Bernoulli sum does not imply unbounded outflows is the classical Bondi solution for spherical accretion flows.

To summarize the present status of the Bernoulli sum described above, at least, the positivity of the sum does not seem to be a genuine property of ADAFs. The sign may depend on various conditions such as dissipation mechanisms (viscous or resistive), the presence of energy transport in a fluid (convection, conduction etc.) and the effects of boundaries. However, it is still controversial whether the sign of the Bernoulli sum can be a direct indicator of the presence of unbounded outflows.

It seems natural to believe that the sum is everywhere zero (the asymptotic value at infinity) in a fluid, as far as the radiation cooling and other mechanisms of energy

redistribution in the fluid are completely negligible, since then all the dissipated energy remains as thermal energy of each fluid element and is merely advected down the flow (i.e., the flow is completely advective). Therefore, a non-zero Bernoulli sum should indicate the intervention of some mechanism of energy redistribution within a non-radiative flow. We may expect convection, conduction or fluid viscosity as such mechanisms of redistribution.

In the present paper, we will derive an analytic model that describes non-radiating accretion flows accompanied by up or down winds in a global magnetic field, based on our previous treatment, K00. Although the main motivation of the present work is the desire to improve broadband spectral fittings to low-luminosity active galactic nuclei (LLAGNs) and normal galaxies (e.g. for Sgr A*, see Kino, Kaburaki & Yamazaki 2000; Yamazaki, Kaburaki & Kino 2001), the resulting solution will be useful to discuss the issues about the Bernoulli sum discussed above.

2. SIMPLIFYING ASSUMPTIONS

A schematic drawing of the global configuration presumed in the present consideration is given in Figure 1 of K00. The viscosity of accreting plasma is completely neglected to clarify the role of magnetic field. An asymptotically uniform magnetic field is vertically penetrating the accretion disk and is twisted by the rotational motion of the plasma. Owing to the Maxwell stress of this twisted magnetic field, a certain fraction of the angular momentum of accreting plasma is carried out to infinity, and this fact enable the plasma gradually infall toward the central black hole.

As shown in Appendix of K00, the geometry of accretion flows is very essential in specifying their physical properties. For example, the ADAF solution follows almost automatically from the assumption of constant opening angle of the disk (i.e., $\Delta = $ const.,

where Δ is the half-opening angle), which respects the spherical nature of the gravity. In this sense, spherical polar coordinates (r, θ, φ) are convenient for the discussion of ADAFs. For the relevant physical quantities, we follow the notation of K00 unless specified otherwise.

Further simplifying assumptions made in K00 in obtaining the no-wind ADAF solution from the set of resistive MHD equations were those of, i) stationarity $(\partial/\partial t = 0)$, ii) axisymmetry $(\partial/\partial\varphi = 0)$, iii) geometrically thin disk, iv) weakly resistive disk, v) dominance of midplane, and vi) no wind. Among these assumptions, only the last one will be removed in the present paper in order to obtain the ADAF solution including winds.

The third assumption means that $\Delta \ll 1$, and is most effective to simplify the basic equations. It has been shown in K00 that the presence of an external magnetic field $\mathbf{B}_0$ guarantees the realization of such a thin disk even in a hot ADAF situation. Reflecting this localized structure, we introduce an angular variable $\eta = (\theta - \pi/2)/\Delta$. Then it becomes clear that a differentiation with respect to θ gives rise to a quantity of order Δ^{-1} $(\partial/\partial\theta = \Delta^{-1}\partial/\partial\eta)$. We can also safely approximate in the disk as $\sin\theta \simeq 1$ and $\cos\theta \simeq 0$.

The statement that a disk is weakly resistive implies that the "characteristic" magnetic Reynolds number $\Re$, whose definition will be introduced later, is large in the sense that $\Re^2(r) \gg 1$ in the disk except near its inner edge $r_{\rm in}$ where $\Re(r_{\rm in}) = 1$. Actually, the terms of $\mathcal{O}(\Re^{-2})$ were neglected in K00. In these situations, the externally given magnetic field $\mathbf{B}_0$ is largely stretched by the rotational and infalling motion of the accreting plasma so that the deformation in the disk becomes much larger than the seed field (if we divide the total magnetic field in the form $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$, then $|\mathbf{b}| \gg |\mathbf{B}_0|$). The definition of the disk's outer edge is the radius within which the deformation of the poloidal magnetic field becomes noticeable (i.e., $|\mathbf{b}_{\rm P}| \sim |\mathbf{B}_0|$).

The assumption v) naturally follows from the consideration that, since majority of

matter is concentrated around the midplane of the accretion disk, its physical properties should be controlled mainly by this part of the disk. Therefore, we may seek the approximate solution that is accurate near the midplane even if making a sacrifice of the accuracy at its upper and lower surfaces. According to this spirit, we ignore the quantities that are proportional to $\tanh^2 \eta$ since $\tanh^2 \eta \ll 1$, $\mathrm{sech}^2 \eta$.

The last condition in the above list is, in fact, not indispensable in obtaining a resistive ADAF solution in a global magnetic field (or resistive ADAF solution). Indeed, an accretion flow accompanied by a converging flow (or down wind) toward the disk surface has been obtained by the present author (Kaburaki 1987) omitting this assumption. It was imposed again in K00, however, in order to understand the energy budget clearly and to firmly establish a basic analytic model in the resistive ADAF regime. This condition actually consists of two equations, $v_\theta = 0$, and $j_\theta = 0$ for consistency. The former results in an r-independent mass accretion rate $\dot{M}$, and the latter specifies the radial dependence of the toroidal magnetic field as $b_\varphi \propto 1/r$ since $j_\theta = -(c/4\pi r)\, \partial(rb_\varphi)/\partial r$ under the assumption of ii).

In relation to the no-wind resistive ADAF solution obtained in K00, it should be stressed that the result actually contains the effects of finite resistivity to the first order in the smallness parameter $\Re^{-1}$. The ratio of poloidal to toroidal magnetic field is small (i.e., $b_r/b_\varphi \sim \Re^{-1}$) reflecting the strong twisting of the poloidal seed field by the rotational motion. This ratio is maintained by the balance between this twisting and untwisting by resistive diffusion. The thin disk structure of the flow is maintained by the vertical force balance between the magnetic pressure of b_φ toward the equatorial plane and the opposing gas pressure in the disk. Reflecting this plasma enhancement in the disk, the gas pressure and density in the disk are the quantities of order $\Re^2$. The finite thickness of the disk itself is also a consequence of non-zero resistivity (indeed, $\Delta \propto \Re^{-1}$). The infall velocity is

small in the sense that $v_r/v_\varphi \sim \Re^{-1}$. Since the magnetic Reynolds number is a function of r, the solution does not have self similarity (i.e., in addition to the presence of angular dependences, both the ratios b_r/b_φ and v_r/v_φ vary with the radial distance).

3.   REMOVAL OF NO-WIND CONDITION

In order to obtain a more general form of the resistive ADAF solution, we remove here the no-wind condition from our list of simplifying assumptions. Fortunately, however, it turns out that all but one terms can be omitted finally among the newly appeared terms in the leading order equations in Δ, by the aid of the assumptions iv) and v). We shall check this point below, expecting that the angular dependence of every quantity (see K00, and §4 below) is not affected by the removal of no-wind condition.

The component expressions of the resistive MHD equations simplified under the assumptions i) through iii) are as follows. They are correct to the leading order in the powers of Δ. In deriving these equations, all quantities except b_θ and v_θ (which are of the order of Δ as confirmed by equations [6] and [7]) are regarded as of order unity with respect to Δ.

- **equation of motion**

    r-component

    $$\left(v_r\frac{\partial}{\partial r} + \frac{v_\theta}{\Delta r}\frac{\partial}{\partial \eta}\right)v_r - \frac{v_\varphi^2}{r}$$
    $$= -\frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{GM}{r^2} + \frac{1}{4\pi\rho r}\left[\frac{b_\theta}{\Delta}\frac{\partial b_r}{\partial \eta} - b_\varphi\frac{\partial}{\partial r}(rb_\varphi)\right] \tag{1}$$

    θ-component

    $$p + \frac{1}{8\pi}\left(b_r^2 + b_\varphi^2\right) = \tilde{p}(r) \tag{2}$$

φ-component

$$\left(v_r\frac{\partial}{\partial r} + \frac{v_\theta}{\Delta r}\frac{\partial}{\partial \eta}\right)v_\varphi + \frac{v_\varphi v_r}{r}$$
$$= \frac{1}{4\pi\rho r}\left[b_r\frac{\partial}{\partial r}(rb_\varphi) + \frac{b_\theta}{\Delta}\frac{\partial b_\varphi}{\partial \eta}\right] \tag{3}$$

- **induction equation**

poloidal component

$$\frac{1}{c}\left(v_r b_\theta - v_\theta b_r\right) = -\frac{c}{4\pi\sigma\Delta}\frac{1}{r}\frac{\partial b_r}{\partial \eta} \tag{4}$$

φ-component

$$r^2 b_r\frac{\partial}{\partial r}\left(\frac{v_\varphi}{r}\right) - \frac{\partial}{\partial r}(rv_r b_\varphi) + \frac{c^2}{4\pi\sigma\Delta^2 r}\frac{\partial^2 b_\varphi}{\partial \eta^2} = 0 \tag{5}$$

- **mass continuity**

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\rho v_r) + \frac{1}{\Delta r}\frac{\partial}{\partial \eta}(\rho v_\theta) = 0 \tag{6}$$

- **magnetic flux conservation**

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 b_r) + \frac{1}{\Delta r}\frac{\partial b_\theta}{\partial \eta} = 0 \tag{7}$$

In r-component of the equation of motion, there appear two new terms reflecting the removal of the no wind condition. They are the second term on the left-hand side and the last term on the right. Both of them are dropped, however, from this equation because of the assumption v) since they are proportional to $\tanh^2\eta$. Further, the first term on the left and the third term on the right are dropped, as in the no-wind case, owing to the assumption iv) since $v_r \propto \Re^{-1}$ and $\rho \propto \Re^{-2}$. Thus, the equation becomes finally so simple as

$$\frac{GM}{r^2} = \frac{v_\varphi^2}{r} - \frac{1}{\rho}\frac{\partial p}{\partial r}. \tag{8}$$

There is no new term in equation (2), and the second term on the left is neglected owing to the assumption iv). The resulting equation describes the magnetic confinement

of the disk plasma by b_φ. In φ-component of the equation of motion, the second term on the left and the first term on the right contain v_θ and j_θ, respectively. However, the former vanishes since v_φ is independent of θ and the latter drops because of $\tanh^2 \eta$ dependence. The resulting form is, as before,

$$v_r \frac{\partial(rv_\varphi)}{\partial r} = \frac{b_\theta}{4\pi\rho\Delta} \frac{\partial b_\varphi}{\partial \eta}. \tag{9}$$

The poloidal components of the magnetic induction equation are degenerate reflecting the degeneracy in Maxwell's equations. The second term on the left of this equation can be dropped again by the assumption v), and we have

$$v_r b_\theta = -\frac{c^2}{4\pi\sigma\Delta} \frac{1}{r} \frac{\partial b_r}{\partial \eta} \tag{10}$$

There appears no change in equation (5).

Thus, the only change that cannot be dropped in the set of basic equations describing magnetized ADAFs including winds is the second term on the left-hand side of equation (6), which results in a radius-dependent mass accretion rate. The equation of magnetic flux conservation is the same as before.

4.   SEPARATION OF VARIABLES

As written out in K00, the the set of resistive MHD equations ([1] $\sim$ [4] there) are 8 equations for 8 unknowns (i.e., for ρ, p, $\mathbf{v}$ and $\mathbf{B}$), and hence the set is closed apparently. Other quantities such as $\mathbf{j}$, $\mathbf{E}$, q (the charge density), and T (the temperature) are calculated from the subsidiary equations, (5) and (6) in K00. Actually, however, the above main set is not closed because of the degeneracy in Maxwell's equations. Usually, this point is supplemented by adding one more equation relating the transfer of energy.

Although, in many cases, the polytropic relation is adopted for simplicity as such

an equation, its validity is rather doubtful except in the special cases of adiabatic and isothermal processes. On the other hand, the full equation for the energy transfer, not only in the fluid but also to the surroundings by the radiation, is too complicated to be treated analytically. Here, we would rather let the set being open as in K00 since an open set does not mean the absence of solutions. Actually, it means only that there is no systematic way of solving it.

Instead of adding energy equation, we have put two specific constraints in K00 in order to obtain the ADAF solution without wind. Before entering into the discussion of them, we first separate the variables in the following form being led by the experience in K00.

$$b_r(\xi, \eta) = \tilde{b}_r(\xi) \operatorname{sech}^2 \eta \tanh \eta, \tag{11}$$

$$b_\theta(\xi, \eta) = \tilde{b}_\theta(\xi) \operatorname{sech}^2 \eta, \tag{12}$$

$$b_\varphi(\xi, \eta) = -\tilde{b}_\varphi(\xi) \tanh \eta, \tag{13}$$

$$v_r(\xi, \eta) = -\tilde{v}_r(\xi) \operatorname{sech}^2 \eta, \tag{14}$$

$$v_\theta(\xi, \eta) = \tilde{v}_\theta(\xi) \tanh \eta, \tag{15}$$

$$v_\varphi(\xi, \eta) = \tilde{v}_\varphi(\xi), \tag{16}$$

$$p(\xi, \eta) = \tilde{p}(\xi) \operatorname{sech}^2 \eta, \tag{17}$$

$$\rho(\xi, \eta) = \tilde{\rho}(\xi) \operatorname{sech}^2 \eta, \tag{18}$$

$$T(\xi, \eta) = \tilde{T}(\xi), \tag{19}$$

$$j_r(\xi, \eta) = -\tilde{j}_r(\xi) \operatorname{sech}^2 \eta, \tag{20}$$

$$j_\theta(\xi, \eta) = \tilde{j}_\theta(\xi) \tanh \eta, \tag{21}$$

$$j_\varphi(\xi, \eta) = -\tilde{j}_\varphi(\xi) \operatorname{sech}^4 \eta, \tag{22}$$

$$E_r(\xi, \eta) = \tilde{E}_r(\xi) \operatorname{sech}^2 \eta, \tag{23}$$

$$E_\theta(\xi, \eta) = \tilde{E}_\theta(\xi) \, \text{sech}^2\eta \tanh \eta, \tag{24}$$

$$E_\varphi(\xi, \eta) = \tilde{E}_\varphi(\xi) \, \text{sech}^4\eta, \tag{25}$$

where the radial coordinate is normalized by a reference radius r_0 as $\xi = r/r_0$. In the problems of accretion in a asymptotically uniform magnetic field, it is natural to choose r_{out} as r_0. The sign of $\tilde{v}_\theta(\xi, \eta)$ is chosen for positive $\tilde{v}_\theta$ to correspond to an up wind (outflow) from the disk surfaces.

The set of basic equations are now rewritten as the set of ordinary differential equations for the radial part functions:

$$\frac{\tilde{v}_\varphi^2}{r} = \frac{1}{\tilde{\rho}} \frac{d\tilde{p}}{dr} + \frac{GM}{r^2}, \tag{26}$$

$$\tilde{p} = \frac{\tilde{b}_\varphi^2}{8\pi}, \tag{27}$$

$$\tilde{v}_r \frac{dl}{dr} = \frac{1}{4\pi\Delta} \frac{\tilde{b}_\theta \tilde{b}_\varphi}{\tilde{\rho}}, \tag{28}$$

$$\tilde{v}_r \tilde{b}_\theta = \frac{c^2}{4\pi\sigma\Delta} \frac{\tilde{b}_r}{r}, \tag{29}$$

$$\tilde{b}_r \left[r^2 \frac{d\Omega}{dr} \right] - \frac{d}{dr}(r\tilde{v}_r\tilde{b}_\varphi) + \frac{c^2}{2\pi\sigma\Delta^2} \frac{\tilde{b}_\varphi}{r} = 0, \tag{30}$$

$$\frac{\tilde{v}_\theta}{\tilde{v}_r} = \Delta \left[r \frac{d}{dr} \ln(r^2 \tilde{\rho} \tilde{v}_r) \right], \tag{31}$$

$$\frac{\tilde{b}_\theta}{\tilde{b}_r} = \frac{\Delta}{2} \left[r \frac{d}{dr} \ln(r^2 \tilde{b}_r) \right], \tag{32}$$

where we have defined $l \equiv r\tilde{v}_\varphi$ and $\Omega \equiv \tilde{v}_\varphi/r$. The angular dependences of the relevant quantities are mutually consistent within the assumption v).

In addition to the above set, the subsidiary equations are

$$\tilde{T} = \frac{\bar{\mu}}{R} \frac{\tilde{p}}{\tilde{\rho}}, \tag{33}$$

$$\tilde{j}_r = \frac{c}{4\pi\Delta} \frac{\tilde{b}_\varphi}{r}, \tag{34}$$

$$\tilde{j}_\theta = \frac{c}{4\pi} \; \frac{1}{r} \frac{d}{dr}(r\tilde{b}_\varphi), \tag{35}$$

$$\tilde{j}_\varphi = \frac{c}{4\pi\Delta} \; \frac{\tilde{b}_r}{r}, \tag{36}$$

$$\tilde{E}_r = -\frac{\Delta}{2} \frac{d}{dr}(r\tilde{E}_\theta), \tag{37}$$

$$\tilde{E}_\theta = \frac{1}{c} \; (\tilde{v}_r\tilde{b}_\varphi - \tilde{v}_\varphi\tilde{b}_r), \tag{38}$$

$$\tilde{E}_\varphi = \frac{1}{c} \; \left(\tilde{v}_r\tilde{b}_\theta - \frac{c^2}{4\pi\sigma\Delta} \; \frac{\tilde{b}_r}{r} \right) = 0, \tag{39}$$

where R is the gas constant, $\bar{\mu}$ is the mean molecular weight, and $\tilde{E}_\varphi$ should vanish owing to the assumption ii).

5.   IRAF CONDITIONS

As mentioned in the previous section, two specific constraints have been placed in obtaining the solution in K00. Then, the solution has been retrospectively shown to be fully advective. Thus, the constraints have taken the place of energy equation, and selected the ADAF solution among others. Therefore, they may be called the ADAF conditions.

We shall adopt the same constraints also in the present paper in order to select the corresponding specific kind of solution to the above set of resistive MHD equations. In view of the resulting solution, however, it seems to be needed to extend the concept of ADAF here. The essence of the conventional ADAF is in that the flow is a very ineffective emitter of radiation field, rather than in the point that internal energy is transported mainly in the form of advection. Such a character is therefore better specified by the term, inefficiently-radiating accretion flow or "IRAF". In such a flow, thermal energy may in general be transported by convection or by conduction as well as by advection. Therefore, we call hereafter the specific constraints the IRAF conditions.

The first of them is

$$
\begin{aligned}
\alpha \ &\equiv \ -\frac{1}{\rho}\frac{\partial p}{\partial r} \ \bigg/ \ \frac{GM}{r^2} \\
&= \ -\frac{1}{\tilde{\rho}}\frac{d\tilde{p}}{dr} \ \bigg/ \ \frac{GM}{r^2} = \text{const.} \ (< 1).
\end{aligned}
\tag{40}
$$

Then, equation (8) or (26) results in a reduced Keplerian rotation of the form

$$
v_\varphi = (1 - \alpha)^{1/2} \ v_\mathrm{K}(r),
\tag{41}
$$

where $v_\mathrm{K}(r) \equiv (GM/r)^{1/2}$ is the Kepler velocity. The first IRAF condition requires that the pressure gradient term, which appears in the equation of radial force balance (8) together with the centrifugal force density, behaves exactly in the same way as the gravitational force density (i.e., they have the same r- and θ-dependences). In general, however, the former is determined as a complicated consequence of the thermal processes such as dissipative heating, radiative cooling and advective cooling, and hence may have different r- and θ-dependences from the latter. In spite of this affair, equation (40) demands the pressure gradient to adjust itself to follow the form of the gravity. Here, one can see the dominance of the gravity over the thermal processes in the IRAF solution.

The second IRAF condition requires that

$$
\begin{aligned}
\beta \ &\equiv \ \frac{1}{b_r}\frac{\partial}{\partial r}(rv_r b_\varphi) \ \bigg/ \ r^2 \frac{\partial}{\partial r}\left(\frac{v_\varphi}{r}\right) \\
&= \ \frac{1}{\tilde{b}_r}\frac{d}{dr}(r\tilde{v}_r \tilde{b}_\varphi) \ \bigg/ \ r^2 \frac{d\Omega}{dr} = \text{const.} \ (< 1)
\end{aligned}
\tag{42}
$$

Similarly to the first condition, the gravity that specifies the rotation law in the denominator determines the degree of magnetic field twist as expressed by the numerator. In general, however, the latter is determined by a complicated consequence of the electrodynamic processes and hence may have different r-, and θ-dependences from the former. Here, we see the dominance of the gravity again, this time, over the electrodynamic processes.

Then, equation (5), or (30) reduces to

$$\tilde{b}_r = -\frac{c^2}{4\pi(1-\beta)\sigma\Delta^2} \left[r^3\frac{d\Omega}{dr}\right]^{-1} \tilde{b}_\varphi. \qquad (43)$$

Substituting equation (41) for Ω, we obtain

$$\frac{\tilde{b}_\varphi}{\tilde{b}_r} = \frac{3\pi(1-\beta)\Delta^2\sigma}{c^2} (1-\alpha)^{1/2}l_{\mathrm{K}} \equiv \Re(r), \qquad (44)$$

where $l_{\mathrm{K}} \equiv \sqrt{GMr}$. This ratio, which is specified by the reduced-Keplerian angular-momentum distribution, is adopted as the characteristic magnetic-Reynolds number of this solution. It must be noted in this regard that the actual magnetic Reynolds number that is defined by the ratio of the convection to diffusion terms in the magnetic induction equation is everywhere and always 1 since, in a stationary state, they are balanced exactly by each other.

6. SOLUTION WITH WINDS

Although a fully advective solution has been automatically obtained in K00 starting from the toroidal field of the form $\tilde{b}_\varphi \propto \xi^{-1}$, which is the consequence of $j_\theta = 0$, we cannot use this relation here in obtaining a more general situation with non-zero v_θ and j_θ. Instead, we start from the following form for the poloidal magnetic field,

$$\tilde{b}_r(\xi) = B_0 \, \xi^{-(3/2-n)}, \qquad (45)$$

where the parameter n specifies the strength of a wind with $n = 0$ corresponding to the no wind case.

Substituting the above expression into equations (44) and (32), we obtain

$$\tilde{b}_\varphi(\xi) = \Re_0 B_0 \, \xi^{-(1-n)}, \qquad (46)$$

$$\Re_0 = \frac{3\pi(1-\alpha)^{1/2}(1-\beta)\sigma\Delta^2}{c^2} l_{\mathrm{K}0}, \qquad (47)$$

and

$$\tilde{b}_\theta(\xi) = \frac{2n+1}{4} \, \Delta B_0 \, \xi^{-(3/2-n)}, \tag{48}$$

respectively. Then, it follows from equation (27) that

$$\tilde{p}(\xi) = \frac{\Re_0^2 B_0^2}{8\pi} \, \xi^{-2(1-n)}. \tag{49}$$

The subscript 0 is referred to each quantity at r_0.

Combining the expressions for $\tilde{b}_r$ and $\tilde{b}_\theta$, we obtain from equation (29)

$$\tilde{v}_r(\xi) = \frac{3(1-\alpha)^{1/2}(1-\beta)}{2n+1} \, \frac{v_{\mathrm{K0}}}{\Re_0} \, \xi^{-1}, \tag{50}$$

and further from this result and equation (31),

$$\tilde{v}_\theta(\xi) = \frac{6(1-\alpha)^{1/2}(1-\beta)n}{2n+1} \, \frac{\Delta v_{\mathrm{K0}}}{\Re_0} \, \xi^{-1}. \tag{51}$$

The density is calculated from equation (28) as

$$\tilde{\rho}(\xi) = \frac{(2n+1)^2}{24\pi(1-\alpha)(1-\beta)} \, \frac{\Re_0^2 B_0^2}{v_{\mathrm{K0}}^2} \, \xi^{-(1-2n)} \tag{52}$$

With the above expressions for various quantities, we can determine first β from the second IRAF condition (42) and then α from the first condition (40) as

$$\alpha = \frac{2}{3}(1-n), \quad \beta = \frac{2}{3}(1-n). \tag{53}$$

Therefore, the final forms of the above tentative expressions are

$$\Re(\xi) = \Re_0 \, \xi^{1/2}, \quad \Re_0 = \left(\frac{2n+1}{3}\right)^{3/2} \frac{3\pi\sigma\Delta^2 l_{\mathrm{K0}}}{c^2} \tag{54}$$

$$\tilde{v}_r(\xi) = \sqrt{\frac{2n+1}{3}} \, \frac{v_{\mathrm{K0}}}{\Re_0} \, \xi^{-1}, \tag{55}$$

$$\tilde{v}_\theta(\xi) = 2n\sqrt{\frac{2n+1}{3}} \, \frac{\Delta v_{\mathrm{K0}}}{\Re_0} \, \xi^{-1}, \tag{56}$$

$$\tilde{v}_\varphi(\xi) = \sqrt{\frac{2n+1}{3}}\, v_{\text{K0}}\, \xi^{-1/2}, \tag{57}$$

$$\tilde{\rho}(\xi) = \frac{3\Re_0^2 B_0^2}{8\pi v_{\text{K0}}^2}\, \xi^{-(1-2n)}, \tag{58}$$

and the subsidiary equations yield

$$\tilde{T}(\xi) = \frac{\bar{\mu}}{R}\, \frac{v_{\text{K0}}^2}{3}\, \xi^{-1}, \tag{59}$$

$$\tilde{j}_r(\xi) = \left(\frac{c}{4\pi\Delta}\right) \frac{\Re_0 B_0}{r_0}\, \xi^{-(2-n)}, \tag{60}$$

$$\tilde{j}_\theta(\xi) = n\left(\frac{c}{4\pi}\right) \frac{\Re_0 B_0}{r_0}\, \xi^{-(2-n)}, \tag{61}$$

$$\tilde{j}_\varphi(\xi) = \left(\frac{c}{4\pi\Delta}\right) \frac{B_0}{r_0}\, \xi^{-(5/2-n)}, \tag{62}$$

$$\tilde{E}_r(\xi) = \tilde{E}_\theta(\xi) = \tilde{E}_\varphi(\xi) = 0. \tag{63}$$

It turns out from the above expressions that the resistive IRAF solution has some characteristic features. First of all, temperature is independent of n and always has the virial value. The radial powers of the all components of velocity are independent of n, while their coefficients depend on n. On the other hand, the powers of other quantities are dependent on n, while their coefficients are independent of n except for $\tilde{b}_\theta$ and $\tilde{j}_\theta$. The vanishing of the electric field reflects one of our implicit assumption that the electric load such as the acceleration of a bipolar jet is completely neglected in obtaining the solution, and hence the total electric power available is spent in the accretion disk.

7.   ENERGY BUDGET

Some global considerations of the physics associated with our accretion disk reveal more information about the present solution. First, from the definition of the disk's inner edge (i.e., $\Re(\xi_{\text{in}}) = 1$) and equation (54), we obtain

$$\xi_{\text{in}} = \Re_0^{-2}, \tag{64}$$

and the condition $r_{\text{in}} \ll r_{\text{out}}$ guarantees $\Re_0^{-2} \ll 1$.

The total magnetic flux penetrating the disk surface can be regarded as being generated by the toroidal current in the disk, since the contribution of the seed field is negligible there. The whole flux closes beyond the outer edge, reducing the magnetic flux in that region. Then, the reduced amount of flux should appear in the narrow central region within the inner edge, with an enhanced strength. Approximating the mean intensity of the central magnetic field by $\tilde{b}_r(\xi_{\text{in}})$, we have

$$\pi r_{\text{in}}^2 \, \tilde{b}_r(r_{\text{in}}) = \int_{r_{\text{in}}}^{r_{\text{out}}} \tilde{b}_\theta \, 2\pi r \, dr, \tag{65}$$

which results in the relation

$$\Delta = \frac{\xi_{\text{in}}^{n+1/2}}{1 - \xi_{\text{in}}^{n+1/2}} \simeq \xi_{\text{in}}^{n+1/2} = \Re_0^{-(2n+1)}. \tag{66}$$

The approximate expression holds because $\xi_{\text{in}}^{n+1/2} \ll 1$ (this point will be confirmed at the end of this section).

Owing to the presence of vertical flows, the mass accretion rate becomes radius dependent like

$$\dot{M}(\xi) = - \int_{-\infty}^{\infty} 2\pi \rho_r r^2 \Delta \, d\eta = \dot{M}_0 \, \xi^{2n}, \tag{67}$$

where

$$\dot{M}_0 = \sqrt{\frac{2n+1}{3}} \, \frac{\Re_0^{-2n} B_0{}^2 r_0^{5/2}}{\sqrt{GM}}. \tag{68}$$

Solving equation (68) for r_0, we obtain

$$r_0 = \left(\frac{3\Re_0^{4n}}{2n+1} \frac{GM\dot{M}_0^2}{B_0{}^4} \right)^{1/5}. \tag{69}$$

It is quite easy to evaluate the Bernoulli sum for the present solution:

$$\text{Be}(\xi, \eta) \equiv \frac{1}{2} v_\varphi{}^2 - \frac{GM}{r} + h = \frac{n}{3} \, v_{\text{K0}}^2 \xi^{-1} \equiv \tilde{\text{Be}}(\xi), \tag{70}$$

where h denotes the specific enthalpy and $h = (5/2)p/\rho$ for any ideal gas. The poloidal velocity does not appear in the kinetic energy term, since its contribution is small compared with that from the toroidal component by $\Re_0^{-2}$. The sum is not a constant along each stream line because the flow is dissipative. For non-zero n, its absolute value increases from zero at infinity as the radius decreases. It is apparent that, within the framework of the present solution, the sign of the Bernoulli sum is indeed a discriminator of wind generation since it directly reflects the sign of n: i.e., according to whether it is positive, zero, or negative we have up wind, no wind, or down wind, respectively. As discussed in §1, the appearance of non-zero sum may indicate the intervention of some other mechanism of energy transport than advection. This point becomes clear soon.

The local energy budget is as follows. The Joule heating rate is calculated as

$$
\begin{aligned}
q_J^+(\xi, \eta) &= \frac{\mathbf{j}^2}{\sigma} \sim \frac{j_r^2}{\sigma} \\
&= \frac{2n+1}{16\pi} \, \Re_0^{2n+1} \, \frac{GM\dot{M}_0}{r_0^4} \, \xi^{-2(2-n)} \mathrm{sech}^4 \eta,
\end{aligned}
\tag{71}
$$

where $c^2/\sigma\Delta^2$ and B_0^2 have been eliminated by the aid of equations (54) and (68). The advection cooling, on the other hand, is

$$
\begin{aligned}
q_{\mathrm{adv}}^-(\xi, \eta) &\equiv \mathrm{div}(h\rho\mathbf{v}) - (\mathbf{v} \cdot \mathrm{grad})p \\
&= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 h\rho v_r) + \frac{1}{r\Delta}\frac{\partial}{\partial \eta}(h\rho v_\theta) - v_r\frac{\partial p}{\partial r} - \frac{v_\theta}{r\Delta}\frac{\partial p}{\partial \eta} \\
&= \frac{4n+1}{16\pi} \, \Re_0^{2n+1} \, \frac{GM\dot{M}_0}{r_0^4} \, \xi^{-2(2-n)} \, \mathrm{sech}^4 \eta,
\end{aligned}
\tag{72}
$$

where the last term in the second line should be dropped on account of the assumption v).

It is evident, therefore, that the advective cooling is in general not balanced by the Joule heating alone. This means that there must be some additional heating that have not been mentioned explicitly, so that the energy balance in a stationary state should in fact be

$$
q_J^+ + q_{\mathrm{add}}^+ = q_{\mathrm{adv}}^-
\tag{73}
$$

since radiation cooling is negligible ($q_{\mathrm{adv}}^{-} \gg q_{\mathrm{rad}}^{-}$) also in the present solution. From the above expressions for q_{J}^{+} and q_{adv}^{-}, we have

$$q_{\mathrm{add}}^{+} = \frac{n}{8\pi} \, \Re_0^{2n+1} \, \frac{GM\dot{M}_0}{r_0^4} \, \xi^{-2(2-n)} \mathrm{sech}^4 \eta. \tag{74}$$

This is what has been suggested just above by the appearance of a non-zero Bernoulli sum.

Plausible examples of such additional heating mechanisms may be viscous heating (q_{vis}^{+}, as is popular in the standard and viscous ADAF theories), convection heating (q_{conv}^{+}, e.g., Narayan, Igumenshchev, & Abramowicz 2000; Quataert & Gruzinov, 2000), conduction heating (q_{cond}^{+}) and so on. Among these examples, the first one should always be positive, while the others may be negative as well. The above result for q_{add}^{+} means that we have up wind, no wind or down wind according to whether the additional heating rate is positive, zero or negative, respectively. In order to keep the global energy budget in a disk, the role of these additional mechanisms should be the redistribution of locally available gravitational energy.

Finally, we derive the restriction on the rage of the wind parameter n. Upper limit for n is derived from the radial dependence of ρ in equation (58). Since the density should decrease outward, we have $n < 1/2$. The most stringent lower limit for n is obtained if we require that the advective cooling should be really cooling (i.e., $-1/4 < n$ when $q_{\mathrm{adv}}^{-} > 0$). This is equivalent to the requirement that entropy should decrease outward, because in a stationary state we can show the equality $q_{\mathrm{adv}}^{-} = \rho T (\mathbf{v} \cdot \mathrm{grad}) s$, where s is the specific entropy. Thus, we have obtained

$$-\frac{1}{4} < n < \frac{1}{2}. \tag{75}$$

This inequality guarantees $\xi_{\mathrm{in}}^{n+1/2} \ll 1$ as far as $\xi_{\mathrm{in}} \ll 1$.

8. SUMMARY AND CONCLUSION

We have extended our former analytic solution for ADAF in a global magnetic field to include winds from the disk surface. The solution contains one parameter n $(-1/4 < n < 1/2)$ that specifies the sense and strength of the winds. According to whether n is positive, zero or negative, we have up wind, no wind or down wind, respectively, with the wind strength increased according to its absolute value.

The following physically different statements are all equivalent mathematically, and represent the criterion for the generation of up winds (i.e., $n > 0$).

(1) Poloidal magnetic field in the disk decreases with radius slower than $r^{-3/2}$.

(2) Rotational velocity of the disk is sufficiently large: i.e., $v_\varphi > (1/\sqrt{3})v_\mathrm{K}$.

(3) Pressure gradient is sufficiently small compare with gravity: i.e., $\alpha < 2/3$.

(4) The Bernoulli sum in the disk is positive.

(5) There are other mechanisms of heating that has not been treated explicitly in the present consideration.

Judging from the radial dependence of the poloidal magnetic field, the converging flow of Kaburaki (1978) corresponds to the extreme case of $n = -1/4$ (down wind). Although the poloidal field of the Blandford-Payne solution (1982) formally corresponds to the case of $n = 1/4$ (up wind), the correspondence should not be taken so seriously because their self-similar solution is somewhat artificial and does not belong to the same class as the solution derived here. The second statement above suggests that the winds are centrifugally driven, and the third, that strong winds apt to appear in high pressure environments as discussed by Fabian & Rees (1995).

The fourth and fifth statements are closely related. The Bernoulli sum works as a discriminator of wind generation, at least within the framework of present treatment. The appearance of non-zero sum seems to indicate the intervention of some other mechanisms of

energy transport than advection. These additional processes act to redistribute the locally available gravitational energy, within a disk. It may be heat conduction or convection that carries thermal energy from inner regions to outer . Another important possibility may be the viscosity that has been completely neglected in this paper for simplicity, but is generally known to have such a redistribution effect (e.g., Frank, King, & Raine 1992). In either case, a more satisfactory solution should be obtained by including each process explicitly in the treatment, and it is of course beyond the scope of this paper.

The wind velocity described by our solution is sub-critical even if it originates from an inner portion of the accretion disk, since $\tilde{v}_\theta/v_K \sim \Delta/\Re(r)$ ($\Delta \ll 1$ and $\Re^{-1}(r) < 1$), and the sound and Alfvén velocities are always of the order of the Kepler velocity, i.e., $V_S(r) \simeq V_A(r) = \sqrt{2/3}\, v_K(r)$ where $V_S^2 \equiv d\tilde{p}/d\tilde{\rho}$ and $V_A^2 \equiv \tilde{b}_\varphi^2/4\pi\tilde{\rho}$. Therefore, the simple collimation of this type of wind from the surface of an accretion disk cannot explain the formation of a relativistic jet. The generation cite of the latter should probably be attributed to the central region within the inner edge of disk. In spite of low velocities expected for winds from the disk surfaces, their effects on the accretion process can be appreciable as seen from the resulting radius-dependent mass accretion rates.

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